or, combining the distortion terms,

$$A_P = A_0\left[1 + \frac{P}{E}\left(2\sigma + \frac{R^2}{R'^2 - R^2}\right)\right] \,. \qquad (2.6)$$

In the limiting case with R'/R effectively infinite this reduces to the simple expression

$$A_P = A_0\left(1 + \frac{2\sigma}{E}\cdot P\right) \,. \qquad (2.7)$$

Equations (2.4) to (2.7) are a useful basis for the development of certain small correction terms which arise in the theory of the similarity and flow methods.

3. The Similarity Method

a) Principle of the method

In normal practice the assemblies for which calibrations are principally required are constructed of steel. The principle adopted in the similarity method is first to determine the ratio of the effective area of the steel piston-cylinder assembly of given type, at a series of pressures, to that of a precisely similar assembly constructed of a material having a substantially different elastic modulus. This procedure determines the difference between the distortion factors of the two assemblies as a function of pressure. A second relation — the quotient of the two distortion factors — is obtained from measurements of the elastic moduli of the two materials. The combination of these results then allows the distortion factor of each assembly to be derived, as a function of pressure, in absolute terms.

b) Ideal theory of the similarity method

In its ideal form the similarity method is extremely simple, and involves no assumption regarding the form of distortion of the assembly when under pressure. In the ideal situation the two materials are regarded as elastically isotropic, with linear stress-strain relationships and identical Poisson's ratios over the range of stress involved. The two assemblies are assumed to be constructed to the same principal dimensions and to have accurately straight and circular pistons and cylinder bores. Ideally, the initial radial separations between the components of the two assemblies should be in inverse ratio to their elastic moduli, although it is found in practice that this condition is not critical. These conditions ensure that, as the distortion changes with increasing pressure, the annular channels between piston and cylinder will remain similar in form and that consequently the pressure distributions along the lengths of the channels will always remain the same for the same total applied pressure.

If these assumptions are realised the distortion terms in the expressions for the effective areas will remain in a fixed numerical ratio as the pressure is varied. In other words the effective areas A_P and B_P of the two assemblies at the applied pressure P may be written in the form,

$$A_P = A_0[1 + \lambda_A f(P)]\,;\; B_P = B_0[1 + \lambda_B f(P)] \qquad (3.1)$$

where λ_A, λ_B are constants in inverse ratio to the elastic moduli, and $f(P)$ is a function of the applied pressure of which the form is unknown but is the same in both cases. Bearing in mind that the distortion terms are normally very small compared with unity, the ratio of the areas may be expressed in the form

$$\frac{A_P}{B_P} = \frac{A_0}{B_0}[1 + (\lambda_A - \lambda_B) f(P)] \qquad (3.2)$$

and writing $\lambda_B = k\lambda_A$, where k is a constant, we obtain

$$\frac{A_P}{B_P} = \frac{A_0}{B_0}[1 + (1 - k)\lambda_A f(P)] \,. \qquad (3.3)$$

The ratio A_P/B_P, and consequently the function $(1 - k)\,\lambda_A\, f\,(P)$, may be determined easily and with high precision by simply measuring the loads on the two pistons when the assemblies are balanced against one another and in equilibrium at the same pressure, and carrying out this procedure at a series of pressures over the appropriate range. The quotient, k, of the elastic moduli may be determined by the standard methods for the measurement of elastic constants. It is clear that in the ideal conditions postulated these two procedures suffice to establish the values of the distortion terms $\lambda_A\, f(P)$ and $\lambda_B\, f(P)$ to an accuracy limited only by the sensitivity of the balancing process and the precision to which the elastic constants are known. In general it is found to be the second factor which eventually limits the accuracy attainable, and to obtain the best precision k should evidently differ substantially from unity.

It is of particular interest that the rheological properties of the pressure transmitting fluid — e. g. dependence of coefficient of viscosity upon pressure — are entirely eliminated in the similarity procedure.

In order to simplify further discussion it is useful at this point to anticipate one practical result of the investigation, viz. that in most cases the distortion is very closely represented by a linear function of the applied pressure so that we may normally replace $f(P)$ by P, when the quantities λ_A and λ_B may be regarded simply as pressure coefficients having the dimensions $(\text{pressure})^{-1}$. Thus we may write instead of (3.1), $A_P = A_0\ (1 + \lambda_A P)$ etc., in all but exceptional cases.

c) Effect of departures from the ideal conditions

It would be a somewhat fortunate coincidence if the ideal assumptions were completely realised in a pair of actual metals having a sufficiently large ratio of elastic moduli, and also adequate tensile strengths, to justify their use in practice, and it is necessary to consider to what extent minor departures may be tolerated, or whether reliable correction terms can be developed. Materials showing appreciable elastic anisotropy are hardly worth consideration owing to the greatly increased complexity of the distortion of the system, and the labour of determining the complete set of elastic constants over a wide range of stress. Again, a pronounced departure from a linear stress-strain relation would introduce awkward complications; small departures may be tolerable, subject to a corresponding uncertainty in the value of the elastic modulus. In the case of a moderate difference in the values of the Poisson's ratios, however, it is not difficult to formulate a correction term. This is small and need only be evaluated approximately. For this purpose we make use of the formula (2.4), and express the distortion coefficients in the form $\lambda_A = \theta_A + \varphi_A \ldots$ where

$$\theta_A = (3\,\sigma_{(A)} - 1)/2\,E_{(A)} \ldots \qquad (3.4)$$

and φ_A is that part of λ_A which is explicitly dependent